Let $m=0$.

$$\therefore\ V=\tfrac{1}{3}n^2c^3\left[\tfrac{1}{2}\pi-\sin^{-1}\left(\frac{a}{nc}\right)\right]$$

$$+\frac{a^3}{3n}\log\left(\frac{nc+\sqrt{[n^2c^2-a^2]}}{a}\right)-\tfrac{2}{3}ac\sqrt{[n^2c^2-a^2]}$$

$$=\tfrac{1}{3}c\left[\frac{\pi R^2}{2}-\sin^{-1}\left(\frac{a}{R}\right)+a^3\log\left(\frac{R+\sqrt{[R^2-a^2]}}{a}\right)-2a\sqrt{[R^2-a^2]}\right]$$

$$=\frac{Rh}{3(R-r)}\left[\frac{\pi R^2}{2}-R^2\sin^{-1}(r/R)+\frac{Rr^3}{R-r}\log\left(\frac{R+\sqrt{[R^2-r^2]}}{r}\right)-2r\sqrt{[R^2+r^2]}\right]$$

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

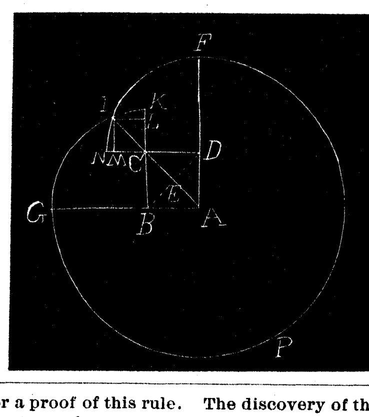
93. **Proposed by RAYMOND D. SMITH, Tiffin, Ohio.**

A barn 20 feet square is standing in a pasture, and a horse is tied to one corner of it with a rope 50 feet long. Over how much land can he graze?

I. Solution by B. F. FINKEL, M. Sc., M. A., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Let $ABCD$ be the barn, side $AB=AD=20$ feet; A the corner to which the horse is tied; and $AF=AG=50$ feet, the length of the rope.

Then $DI=BI=30$ feet; $AC=DB=20\sqrt{2}$ feet; $EI=\sqrt{[DI^2-DE^2]}=\sqrt{[30^2-(10\sqrt{2})^2]}$ feet$+10\sqrt{7}$ feet; $CI=EI-EC=10\sqrt{7}$ feet$-10\sqrt{2}$ feet$=10(\sqrt{7}-\sqrt{2})$ feet; $CL=CM=\sqrt{[CI^2/2]}=\frac{1}{2}CI\sqrt{2}=5(\sqrt{14}-2)$ feet; $KL=CK-CL=10$ feet$-5(\sqrt{14}-2)$ feet$=5(4-\sqrt{14})$ feet; and chord $KI=$chord $IN=\sqrt{[KL^2+IL^2]}$.



$\sqrt{[25(4-\sqrt{14})^2+25^2(\sqrt{14}-2)^2]}$ feet $=10\sqrt{[3(4-\sqrt{14}]}$ feet.

2 arc $IK=\frac{1}{3}(8$ chord $KI-2IL$* $=\frac{1}{3}\{80\sqrt{[3(4-\sqrt{14})]}-20(\sqrt{7}-\sqrt{2})\}$ feet$=\frac{20}{3}\{4\sqrt{[3(4-\sqrt{14})]}-(\sqrt{7}-\sqrt{2})]$ feet.

The area over which the horse can graze$=FAGPF+$sector $FDI+$sector $IBG+$ triangle $DCI+$triangle $BCI=FAGPF+2$ sector $FDI+2$ triangle $DCI=FAGPF+2($quadrant $FDN-$sector $IDN)+2$ triangle DCI.

But area of $FAGPF=\frac{3}{4}\pi AF^2=1875\pi$;

*See *Williamson's Differential Calculus*, pages 64-65, for a proof of this rule. The discovery of this important approximation is due to Huygens. The length of an arc of 30° on a circle of radius 100,000 differs from the true value, assuming $\pi=3.141592$, by about 2 inches. The formula is arc$=\frac{1}{3}(8B-A)$ when B is the chord of half the arc and A is chord ofthe arc.

area of quadrant $FDN=\frac{1}{4}\pi DF^2=225\pi$; and area of sector $IDN : 2\pi DF^2 :: 2\pi DF$: arc IN, or area of sector $IDN=\frac{1}{2}DF\times$ arc $IN=15\{\frac{10}{3}[4\sqrt{3[(4-\sqrt{14})]}-\sqrt{7}+\sqrt{2}]\}$ square feet$=50\{4\sqrt{[3(4-\sqrt{14})}-\sqrt{7}+\sqrt{2}\}$ square feet.

$\therefore$ Area of sector $FDI=225\pi-50\{4\sqrt{[3(4-\sqrt{14})]}-\sqrt{7}+\sqrt{2}\}$.

Area of triangle $DCI=\frac{1}{2}DC\times IM=10\times5(\sqrt{14}-2)=50(\sqrt{14}-2)$ square feet. $\therefore$ The total area over which the horse can graze$=1875\pi+2(225\pi-50\{4\sqrt{[3(4-\sqrt{14})]}-\sqrt{7}+\sqrt{2}\})+2[50(\sqrt{14}-2)]=1875\pi+450\pi-1004\sqrt{[3(4-\sqrt{14})]}-\sqrt{7}+\sqrt{2}+100(\sqrt{14}-2)=2375\pi+100\sqrt{14}-2-4\sqrt{[3(4-\sqrt{14})]}+\sqrt{7}-\sqrt{2}=7249.378$ square feet.

II. Solution by G. I. HOPKINS, A. M., Professor of Mathematics and Physics, High School, Manchester, N. H.; J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pa.; and P. S. BERG, Principal of School, Larimore, N. D.

Let $ABCD$ represent the barn and A the corner to which the horse is tied. Then $FA=AG=50$ feet, and $DF=ID=BI=GB=30$. The area over which he can graze is divided into four parts, viz.: the three-quarters of a circle $AFPGA$, the two sectors GBI and IDF, and the quadrilateral $IBCD$.

$BD=\sqrt{(20^2+20^2)}=20\sqrt{2}$. $\therefore$ $ED=10\sqrt{2}$.

$\therefore$ $CE=\sqrt{[30^2-(10\sqrt{2})^2]}=10\sqrt{7}$.

Area $IBCD$=area $IBD-$area BCD.

$\therefore$ Area $IBCD=10\sqrt{7}\times10\sqrt{2}-200=100\sqrt{14}-200$.

$\therefore$ Area $IBCD=174.1657$ square feet.

$\cos\angle IDE=(10\sqrt{2})/30=.4714$.

$\therefore$ $\angle IDE=61^\circ\ 52'\ 30''$ and $\angle BDA=45^\circ$.

$\therefore$ $\angle IDA=106^\circ\ 52'\ 30''$. $\therefore$ $\angle IDF=73^\circ\ 7'\ 30''$.

Sectors GIC and IDF are equal. $\therefore$ $2\angle 73^\circ\ 7'\ 30''=146^\circ\ 15'=146\frac{1}{4}^\circ$.

Area of circle whose radius is $ID=30^2\pi=900\pi$. $\therefore$ The areas of the two sectors GBI and $IDF=(146\frac{1}{4}/360)\times900\pi=365\frac{5}{8}\pi=365.625\pi$.

Area of $GAFPG=(3\cdot50^2\pi)/4=1875\pi$. $(365.625+1875)\pi=2240.625\pi$.

$\therefore$ Area $GAFPG=7039.1475$ square feet.

$\therefore$ Entire area$=174.1657+7039.1475=7213.3132$ square feet$=26.495$ square rods.

This problem was also solved by G. B. M. Zerr, who got as an answer 7291.9868 square feet; J. Scheffer, his answer being 6889.414 square feet; Fremont Crane, his result being 6351.785 square feet; and B. F. Sine, his result being 7233.292 square feet. Cooper D. Schmitt did not solve it, but referred to a previous solution in the MONTHLY.

94. Proposed by J. A. CALDERHEAD, M. Sc., Professor of Mathematics in Curry University, Pittsburg, Pa.

What rate of income do I realize by purchasing United States 4% bonds at 105 if I sell them in six years at 104?

Solution by CHARLES C. CROSS, Libertytown, Md.; FREMONT CRANE, Sand Coulee, Mont.; HON. JOSIAH H. DRUMMOND, Portland, Me.; and G. B. M. ZERR, Pottstown, Pa.

$.04\times6=24$.

$1.04+.24=1.28$, amount realized on bond.

$1.28-1.05=.23$, amount gained in six years.